Holistic Ice Sheet Modeling: A First-Order Approach

Shrae Ashley
Jarvis Christian College

Keila Vance
Jackson State University

Dr. Malcolm LeCompte
Mentor
Elizabeth City State University

Dr. Terry Hughes
Mentor
University of Maine
Climate Change Institute

Abstract-**The ECSU Summer 2006 Undergraduate Research Experience (URE) Holistic Ice Sheet Modeling Team applied a first-order analytic model of a continental ice sheet, based on balancing gravitational pressure forces against externally applied basal shearing stresses, to derive structural characteristics and infer dynamic behavior of a continental ice sheet. First-order model results were compared to surface elevation and ice thickness data obtained by an aerial ice penetrating RADAR survey of the Antarctic's Byrd Ice Stream (Jezek et. al., 1998) The first order model results were also compared to those of a higher order analytic approximation used to interpret the data (Reusch and Hughes, 2003).**

The first-order model was expanded to include mass balance considerations and a parameterized description of ice basal surface coupling and bed texture roughness and topography as well a side shear, compressive and tensile forces. The expanded first-order model was studied to better understand the dynamic variations in ice sheet and ice stream behavior resulting from changes in ice accumulation and ablation rates that may be possible consequences of Global Climate Warming. Study of the first-order ice sheet model included consideration of its evolution from completely grounded continental sheet to a partially grounded ice stream flowing to its terminus as a calving ice shelf.

I. Introduction

The last million years has been the Quaternary Ice Age. Figure 1 compares the present-day coastline of North America with the coastline 20,000 years ago at the last glacial maximum when an ice sheet extend from Greenland, across Canada into the United States, and with the coastline if the ice sheets now blanketing Greenland and Antarctica were to disappear. Then Memphis in Tennessee would be an ocean port and New England would be an island!

Figure 1. North American continental shorelines defined by sea level changes linked to Quaternary glaciations.

Ice Sheets are known to be dynamic components of Earth's climate machine, and may be the system most responsible for rapid changes in climate and rising sea level. In the first-order treatment presented here, many important physical processes are excluded in order to focus attention on the physical processes that control rapid changes in the unstable outer periphery of ice sheets, and ultimately stability of the interior core as well.

II. Methodology

A simple first-order ice sheet model was derived from simple geometrical consideration of a force balance between gravitationally forced spreading resisted by basal coupling stress due to the ice sheet being frozen to its bed. The model is used to provide a first-order understanding of the thickness of an ice sheet as it spreads across a continental land mass, progressing from sheet-, to stream-, to shelf-flow as described below:

– **Sheet-Flow**: a grounded ice sheet with 100% of its basal surface frozen to its bed

– **Stream-Flow**: a mass of ice, flowing toward the sea whose basal surface is partially afloat and partially frozen to its bed.
– **Shelf-Flow**: a large area of floating Ice.

An Ice sheet is thickest where it is grounded and continues to thin until it is 100% afloat at its grounding line. Once afloat, the ice continues to thin due to the force of gravity and difference in density between ice and water. The first-order ice sheet model was developed from an assumption of balance between gravitational forcing of the ice sheet resisted by basal traction allowing the ice sheet's most dynamic component: the ice stream; to be understood and modeled by the variability of coupling to the bed over which it flows. Figure 2 illustrates the structural elements of the first-order model approach to modeling ice sheets.

Higher order geometrically derived models, accounting for the effects of side shear stress, compression and tension forces and mass balance, can be derived from the simple force balance treatment Basal coupling in all these models is parametrically determined by the Floating Fraction of ice, denoted by the Greek letter Phi (Φ) and defined quantitatively as the ratio of basal water pressure necessary to float the ungrounded fraction of the ice sheet to the overburden pressure of the ice above the bed. It's higher order quantitative definition is:

$$\Phi = P_W/P_I \qquad (1)$$

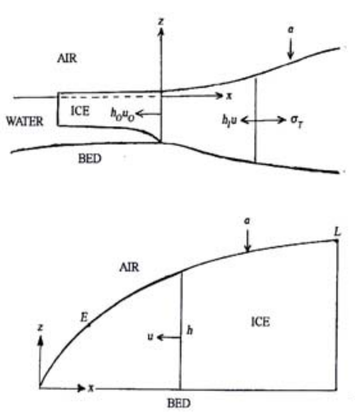


Figure 2: The first-order Ice Sheet model depictions: ice stream terminating in floating ice shelf (upper) and Ice Sheet terminal lobe (lower).

The weight (W) of a column of ice in a continental sheet at its highest point (the "Ice Divide") is a function of its thickness (h_I) above the bed:

$$W = P \bullet h_I \bullet w \qquad (2)$$

Where P is the pressure under a column ice of height (h_I) and w is the width of the ice sheet. The pressure increases with ice depth.

$$P= \rho_I \bullet g \bullet h_I \qquad (3)$$

and it is easily shown from simple geometric considerations that the average pressure through the ice sheet <P> is given by:

$$\langle P\rangle = (\rho_I \bullet g \bullet h_I) \ / \ 2 \qquad (4)$$

The average gravitationally generated pressure <P> would cause the ice to continue to spread until it moved into the ocean and attained a thickness equal to that of floating ice shelf in hydrostatic balance. However, in the simplest case (horizontal bed), the spreading is resisted by the fact that the ice sheet basal surface is frozen to its bed. The resultant basal traction acts to prevent the ice sheet from thinning.

The basal traction force is equal to the basal stress term τ_o, multiplied by the area over which it acts, In others words, τ_o , multiplied by the area over which the ice sheet is frozen to the bed. The area is equal to width (w) times the length (x). This simple balance of force relation is expressed:

$$F(x) = \langle P\rangle \bullet h_I \bullet w - \tau_o \bullet w \bullet x= (\rho_I \bullet g \bullet h_I \bullet h_I \bullet \cdot w)/2 – (\tau_o \bullet w \bullet x) = 0$$
(5)

Canceling like terms leaves an expression for the ice sheet thickness as a function of long distance along the bed (x).

$$\tau_o \bullet x = \rho_I \bullet g \bullet h_I^2 \qquad (6)$$

Or solving for x:

$$x = \rho_I \bullet g \bullet h_I^2 / (2 \bullet \tau_o) \qquad (7)$$

As an ice sheet approaches the water's edge over uneven ground, an increasing amount of the ice basal surface will be uncoupled from the bed and begin to float. This can be reflected in the force balance equation modifying the basal stress term by adding a parameter that changes the area of ice frozen to the bed. The area will be reduced by an amount dependent on variation of the floating fraction of the ice sheet (Φ), a measure of the fraction of the basal surface area over which the ice sheet, now an ice stream, is floating. When the floating fraction becomes equal to 1, the ice is totally afloat and has become an ice shelf of thickness h_o at the grounding line, then

Intuitively, the ice sheet and stream can never be thinner than at the grounding line where it is afloat. As the floating fraction diminishes going toward the ice sheet (where the fraction will be equal to zero), its thickness will increase as the floating fraction decreases.

The ratio of ice thickness at the grounding line (h_o) to its thickness anywhere along the ice stream (h_I) is thus a first-order measure of the floating fraction Φ:

$$\Phi = h_o / h_I \qquad (8)$$

In this case, the first-order force balance equation for an ice stream would be:

$$(\rho_I \cdot g \cdot h_I \cdot h_I\, w/2) - \tau_o \cdot w \cdot x \cdot (1-\Phi) = 0 \qquad (9)$$

resulting in an expression relating thickness of the ice to the distance along the stream:

$$x = \rho_I \cdot g \cdot h_I^2 / (2 \cdot \tau_o \cdot (1-\Phi)) \qquad (10)$$

If we assume a profile for Φ(x) and have a value of the ice thickness of the grounding line, then the first-order relation, $\Phi = h_o/h_I$, provides a means to measure the thickness of the ice along the stream.

III. Data vs. Model Comparison

Figure 3 shows RADAR measurements acquired by an aerial survey (Jezek et. al. 1998) yielding ice thickness data along Byrd Glacier on a flow line from ice shelf to continental interior. Byrd Glacier drains a significant amount of the East Antarctic Ice Sheet through the Trans-Antarctic Mountains into the West Antarctic basin now occupied by the Ross Ice Shelf. The thickness data is used herein as the basis for comparing various first and higher-order estimates of the floating fraction to gain insight into the behavior of the Byrd Glacier prior to its merging with the Ross ice shelf. Ice thickness values were calculated for first-order and higher-order

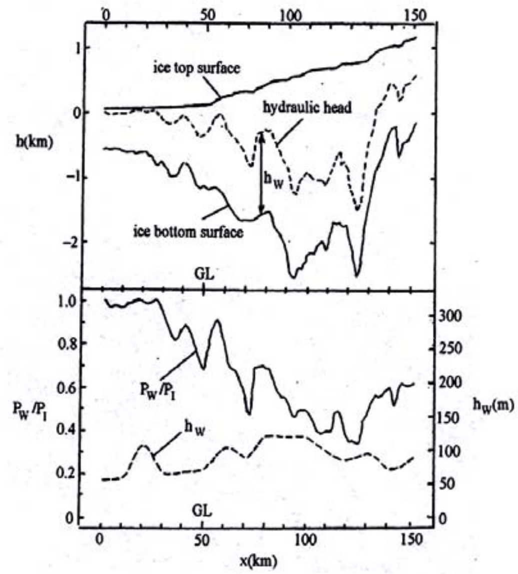
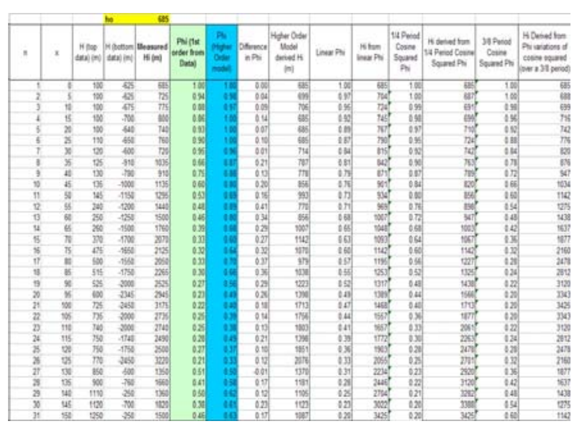


Figure 3: varying surface elevation, bed depth, P_w/P_I and hydraulic head height (h_w) with distance from the grounding line.

IV. Results

Ice thickness (h_I) values along the ice stream flow-line toward the fjord head-wall were tabulated with distance measured from the grounding line. Grounding line thickness (h_o) was derived from the airborne RADAR measurements (Figure 3) of Byrd ice stream and estimated to be 650 meters. The measured h_I and h_o values were used to calculate a "data-derived" Φ trend, depicted in Figure 4. Also shown in Figure 4 are Φ values corresponding to a higher-order ice thickness model wherein:

$$\Phi = P_W / P_I \qquad (11)$$



			ho	685										
n	x	H (top data) (m)	H (bottom data) (m)	Measured Hi (m)	Phi (1st order from Data)	Phi (Higher Order model)	Difference in Phi	Higher Order Model derived Hi (m)	Linear Phi	Hi from linear Phi	1/4 Period Cosine Squared Phi	Hi derived from 1/4 Period Cosine Squared Phi	3/8 Period Cosine Squared Phi	Hi Derived from Phi variations of cosine squared (over a 3/8 period)
1	0	100	-625	685	1.00	1.00	0.00	685	1.00	685	1.00	685	1.00	685
2	5	100	-625	725	0.94	0.98	0.04	699	0.97	704	1.00	687	1.00	688
3	10	100	-675	775	0.88	0.97	0.09	706	0.95	724	0.99	691	0.98	699
4	15	100	-700	800	0.86	1.00	0.14	685	0.92	745	0.98	699	0.96	716
5	20	100	-640	740	0.93	1.00	0.07	685	0.89	767	0.97	710	0.92	742
6	25	110	-650	760	0.90	1.00	0.10	685	0.87	790	0.95	724	0.88	776
7	30	120	-600	720	0.95	0.96	0.01	714	0.84	815	0.92	742	0.84	820
8	35	125	-910	1035	0.66	0.87	0.21	787	0.81	842	0.90	763	0.78	876
9	40	130	-780	910	0.75	0.88	0.13	778	0.79	871	0.87	789	0.72	947
10	45	135	-1000	1135	0.60	0.80	0.20	856	0.76	901	0.84	820	0.66	1034
11	50	145	-1150	1295	0.53	0.69	0.16	993	0.73	934	0.80	856	0.60	1142
12	55	240	-1200	1440	0.48	0.89	0.41	770	0.71	969	0.76	898	0.54	1275
13	60	250	-1250	1500	0.46	0.80	0.34	856	0.68	1007	0.72	947	0.48	1438
14	65	260	-1500	1760	0.39	0.68	0.29	1007	0.65	1048	0.68	1003	0.42	1637
15	70	370	-1700	2070	0.33	0.60	0.27	1142	0.63	1093	0.64	1067	0.36	1877
16	75	475	-1650	2125	0.32	0.64	0.32	1070	0.60	1142	0.60	1142	0.32	2160
17	80	500	-1550	2050	0.33	0.70	0.37	979	0.57	1195	0.56	1227	0.28	2478
18	85	515	-1750	2265	0.30	0.66	0.36	1038	0.55	1253	0.52	1325	0.24	2812
19	90	525	-2000	2525	0.27	0.56	0.29	1223	0.52	1317	0.48	1438	0.22	3120
20	95	600	-2345	2945	0.23	0.49	0.26	1398	0.49	1389	0.44	1566	0.20	3343
21	100	725	-2450	3175	0.22	0.40	0.18	1713	0.47	1468	0.40	1713	0.20	3425
22	105	735	-2000	2735	0.25	0.39	0.14	1756	0.44	1557	0.36	1877	0.20	3343
23	110	740	-2000	2740	0.25	0.38	0.13	1803	0.41	1657	0.33	2061	0.22	3120
24	115	750	-1740	2490	0.28	0.49	0.21	1398	0.39	1772	0.30	2263	0.24	2812
25	120	750	-1750	2500	0.27	0.37	0.10	1851	0.36	1903	0.28	2478	0.28	2478
26	125	770	-2450	3220	0.21	0.33	0.12	2076	0.33	2055	0.25	2701	0.32	2160
27	130	850	-500	1350	0.51	0.50	-0.01	1370	0.31	2234	0.23	2920	0.36	1877
28	135	900	-760	1660	0.41	0.58	0.17	1181	0.28	2446	0.22	3120	0.42	1637
29	140	1110	-250	1360	0.50	0.62	0.12	1105	0.25	2794	0.21	3282	0.48	1438
30	145	1120	-700	1820	0.38	0.61	0.23	1123	0.23	3022	0.20	3388	0.54	1275
31	150	1250	-250	1500	0.46	0.63	0.17	1087	0.20	3425	0.20	3425	0.60	1142

Figure 4. EXCEL spread sheet tabulations of floating fraction and ice thickness (h_I) with distance from the grounding line.

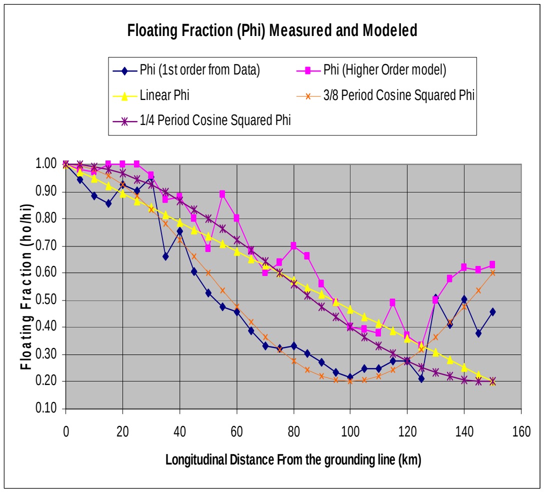


Figure 5: Floating Fractions Φ, for each tabulated case compared to "data-derived" Φ (blue diamonds).

Where P_W is the hydraulic pressure and P_I is the ice overburden pressure.

Floating Fraction trends corresponding to first-order functional variations of Φ were also tabulated and graphed; including a linearly varying case and two periodically varying cases with different spatial frequencies. The period of one cosine squared function was chosen to be maximum at the grounding line decreasing to zero at the fjord head wall. with Values of Φ for each case are graphically depicted in Figure 5.

Figure 6 is a reprise Figure 5 with the linear Φ and the Φ derived from a quarter period square cosine function omitted. The two Φ profiles are easily compared with the blue "data-derived" Φ profile.

As anticipated, the floating fraction, Φ, decreases with distance from the grounding line. Up to 120 km from the grounding line, the first-order model Φ profile seems to correspond more closely to the Φ profile derived from the measured values than do those of the higher-order model.

Figure 7 shows ice thickness (h_I) profiles corresponding to the Φ profiles in Figure 6. The ice stream becomes progressively more grounded approaching the fjord head wall, and therefore becomes thicker. Maximum thickness occurs at approximately 100 km from the grounding line, thereafter the three profiles converge.

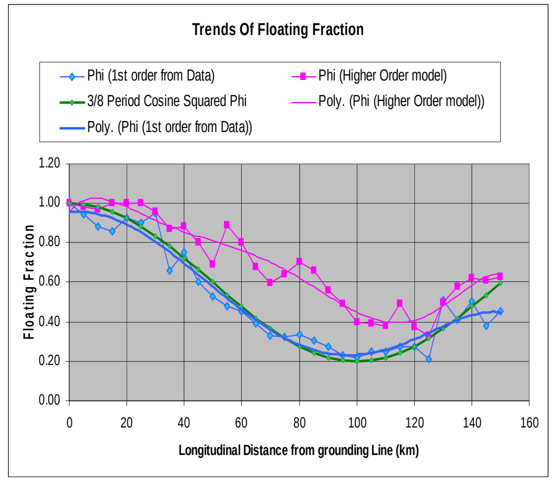


Figure 6. Floating Fraction trends for two models (higher-order and 3/8 period, squared cosine)

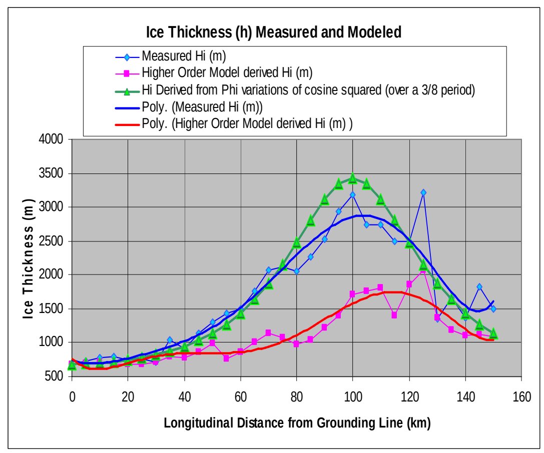


Figure 7: shows the associated first-order thickness (h_I) profiles derived from the data, the higher order model and the periodically varying Φ plotted for 3/8ths of a period.

V. Conclusion

The force-balance approximation is probably weakest near the fjord head-wall, where ice flowing from higher ground is stretched like taffy. Reduction in bed coupling appears as an increase in Phi Φ approaching the fjord headwall. An increase in bed slope may decouple ice from its bed due to the associated increase in the gravitational force component parallel to the bed.

VI. Future Work

In the future one might want to repeat the study using a higher-order holistic first order geometric model treatment that includes: mass balance, side-shearing stresses, topography, compression and tension forces. Yet, even the present, simple derivation of Φ has provided insight into ice stream dynamics.

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