Using RK4 to Model Felix Baumgartner's 38969 m Skydive

Laura[1], Mahatma[2], Sneha[3], Susheel[4],Rodney[5] and Aishat[6]

Austin Peay State University, Clarksville, TN[1]
Winston Salem State University, Winston Salem, NC[2]
Indiana University, Bloomington, IN[3]
Southern Illinois University, Carbondale, IL[4]
Florida A&M University, Tallahassee, FL[5]
Albany State University, Albany, GA[6]

13 July 2017

Unit Testing
Felix Freefall
Felix With Parachute
RK4 Method
Sensitivity Tests
Summary
References

Problem Statement

- Using Python to develop a basic parachute model
- Run simulations to test the sensitivity
- Parameters considered are:
  - Different weights of skydiver
  - Different times
  - Different release heights

Figure: Felix Baumgartner in freefall

Unit Testing

Felix Freefall

Felix With Parachute

RK4 Method

Sensitivity Tests

Summary

References

Outline

Unit Testing

Felix Freefall

Felix With Parachute

RK4 Method

Sensitivity Tests

Summary

References

Unit Test: Freefall vs With Drag

For our unit test, we started at a height of 550 m and a velocity of 10 m/s. Without drag, we compared the total time to the analytical solution of

$$\frac{1}{2}gt^2 + 10t + 550 = 0 \tag{1}$$

and got 11.66 s for both. With drag, we compared the terminal speed to the analytical solution of

$$v_t = \sqrt{\frac{2mg}{\rho A C_d}} \tag{2}$$

and got 33.88 m/s for both. Consequently, we concluded that our basic ball model was valid.

Unit Test: Freefall vs With Drag

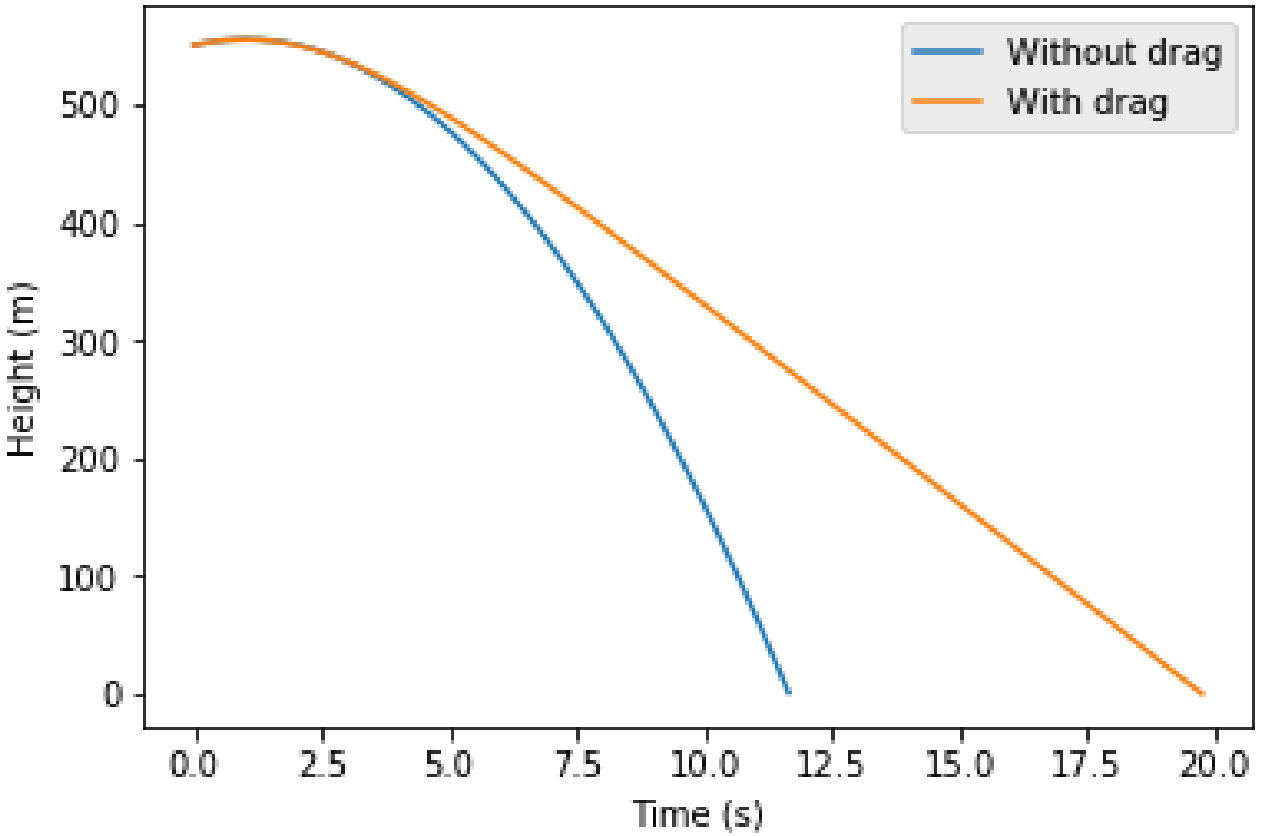

Figure: Projectile motion of ball in freefall vs with drag

Assumptions and Setup for FreeFall

- Assumed the linear drag term was negligible
- Used Newton's gravitational law

$$F_G = -G\frac{Mm}{r^2} \tag{3}$$

  instead of assuming constant
- Used barometric formula [1] to get density as a function of altitude

Felix Freefall

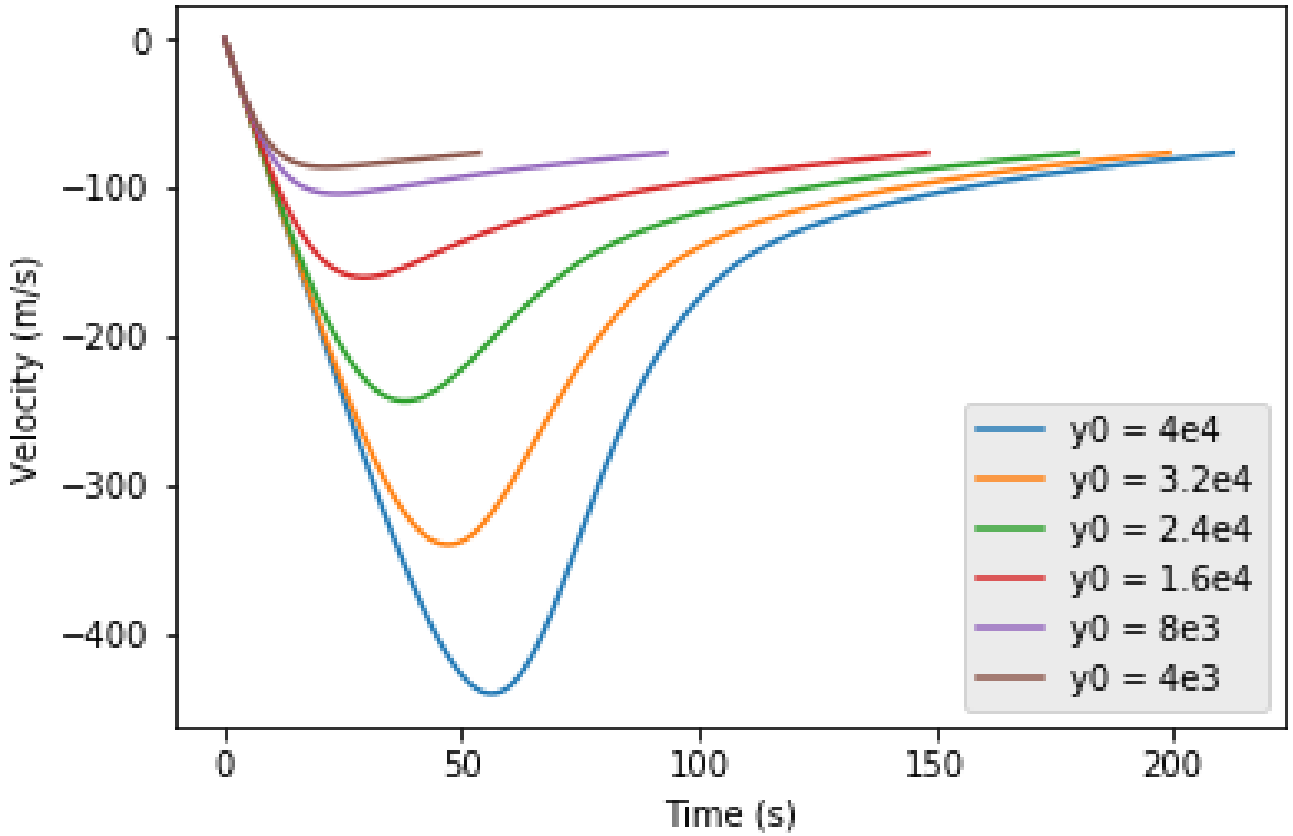

Figure: Velocity vs time for various start heights

Assumptions and Setup for Parachute

- Assumed parachute release was instantaneous
- Assumed drag due to parachute much bigger than drag due to man
- Built in user input for testing various conditions

Unit Testing

Felix Freefall

Felix With Parachute

RK4 Method

Sensitivity Tests

Summary

References

Unit Test

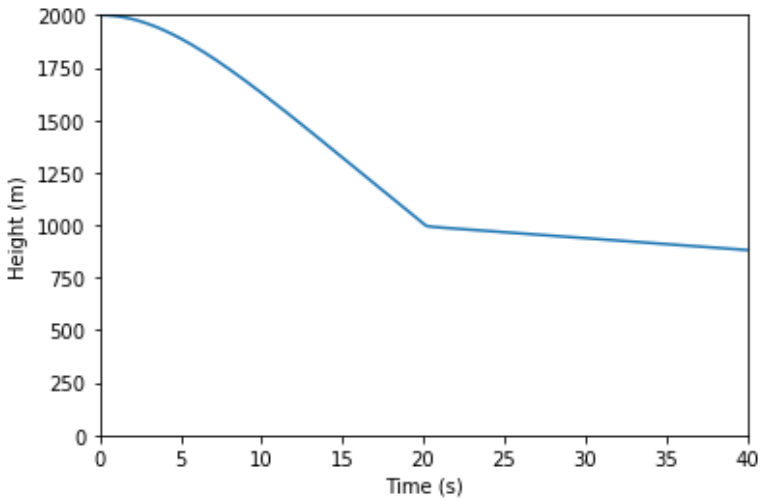

Figure: Position vs time from model

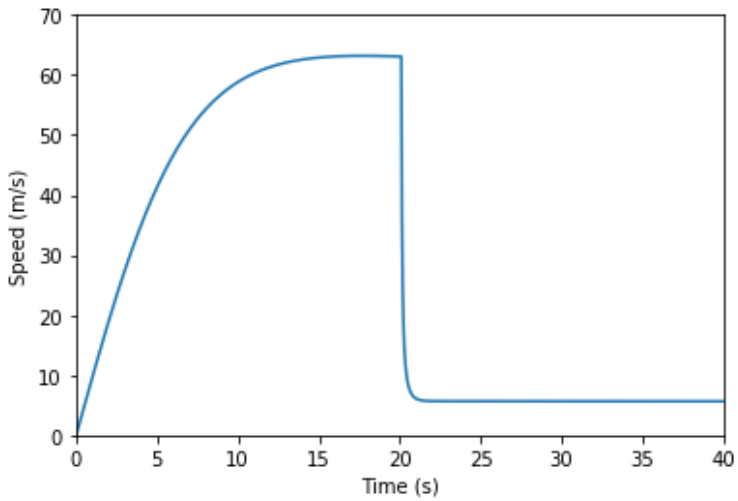

Figure: Velocity vs time from model

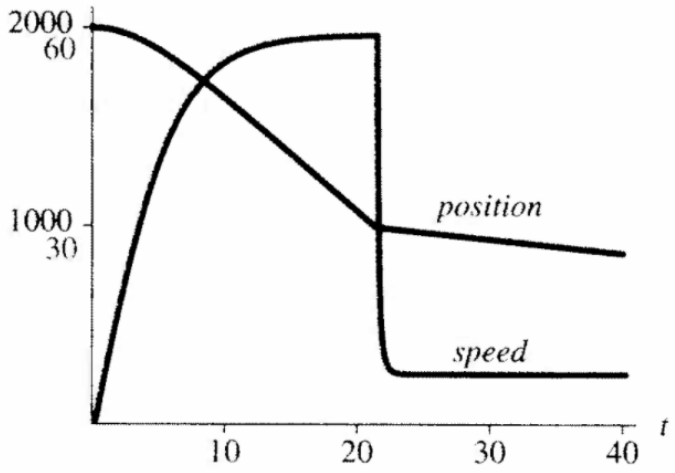

Figure: A. B. Shiflet [2] example of position and velocity vs time

Results

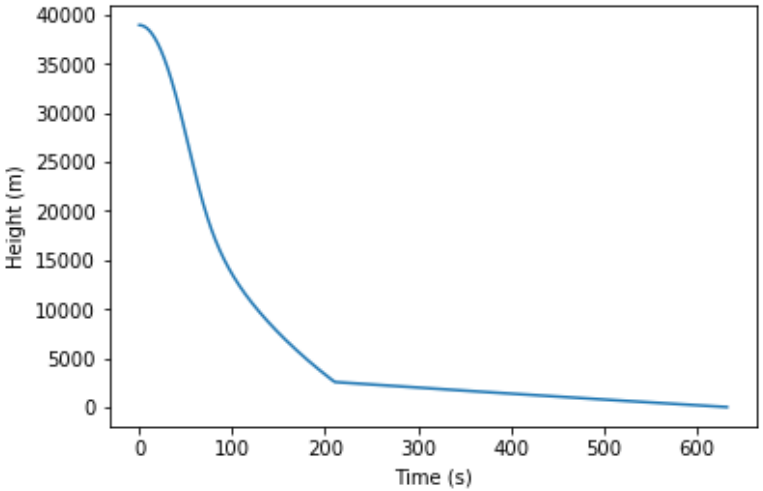

Figure: Position vs time

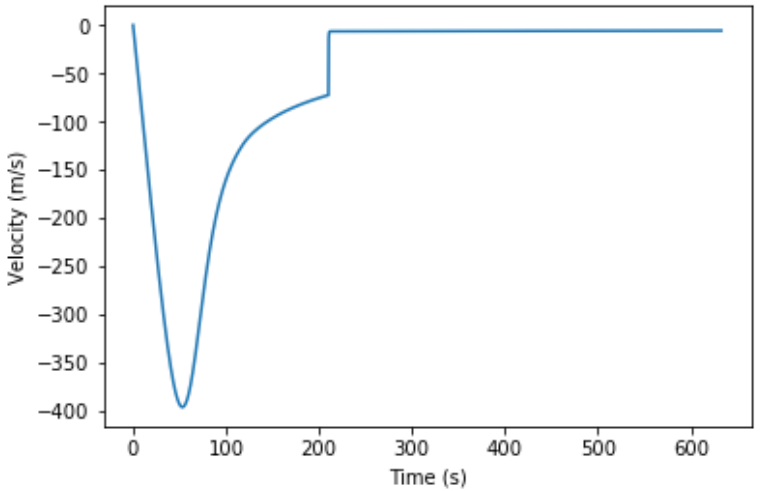

Figure: Velocity vs time

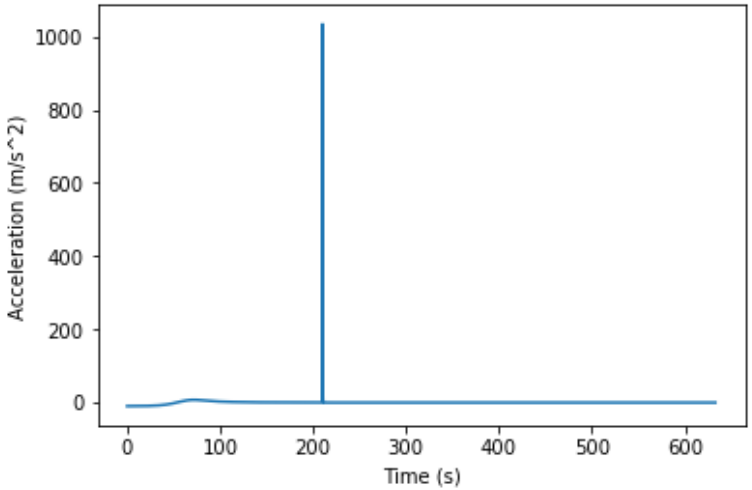

Figure: Acceleration vs time

Result	Given	Model
t_f (s)	549	642
t_R (s)	256	210
v_{max} (m/s)	372	395

Table: Given results vs results of model

```
k1R = v[i,:]
k1V = a[i,:]
tempR = r[i,:] + 0.5*timeStep*k1R
tempV = v[i,:] + 0.5*timeStep*k1V
k2R = tempV
k2V = pm.dragForce(tempR[2], tempV, dragCoefMan, areaMan,
dragCoefPar, areaPar, parIsOpen)/m
k2V[2] = k2V[2] - G*Me/math.pow(tempR[2] + Re,2)
tempR = r[i,:] + 0.5*timeStep*k2R
tempV = v[i,:] + 0.5*timeStep*k2V
k3R = tempV
k3V = pm.dragForce(tempR[2], tempV, dragCoefMan, areaMan,
dragCoefPar, areaPar, parIsOpen)/m
k3V[2] = k3V[2] - G*Me/math.pow(tempR[2] + Re,2)
tempR = r[i,:] + timeStep*k3R
tempV = v[i,:] + timeStep*k3V
k4R = tempV
k4V = pm.dragForce(tempR[2], tempV, dragCoefMan, areaMan,
dragCoefPar, areaPar, parIsOpen)/m
k4V[2] = k4V[2] - G*Me/math.pow(tempR[2] + Re,2)
r[i+1,:] = r[i,:] + timeStep*(k1R + 2.0*k2R + 2.0*k3R +
k4R)/6.0
v[i+1,:] = v[i,:] + timeStep*(k1V + 2.0*k2V + 2.0*k3V +
k4V)/6.0
a[i+1,:] = pm.dragForce(r[i+1,2], tempV,
dragCoefMan, areaMan, dragCoefPar, areaPar, parIsOpen)/m
a[i+1,2] = a[i+1,2] - G*Me/math.pow(r[i+1,2] + Re,2)
```

Sensitivity to Mass

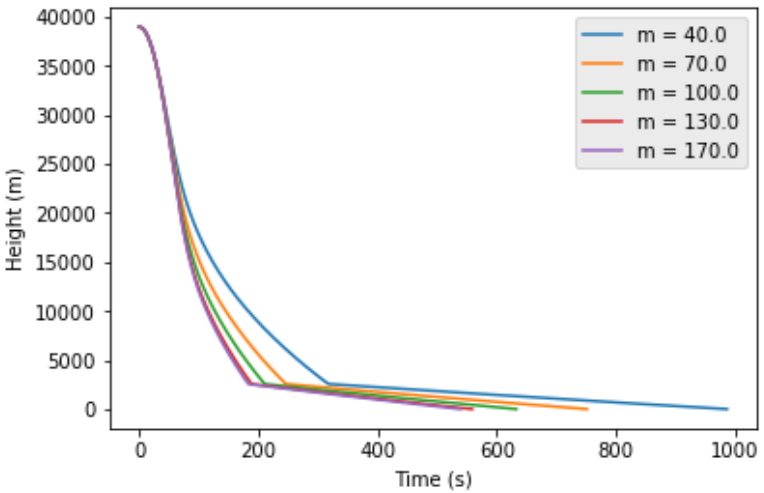

Figure: Position vs time

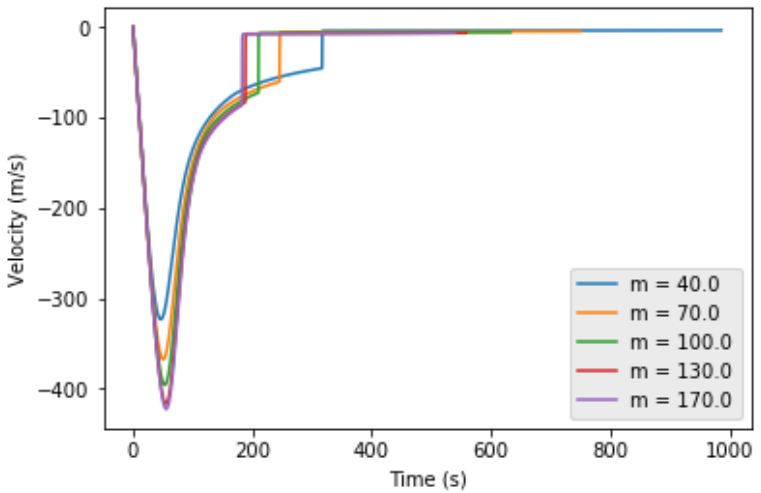

Figure: Velocity vs time

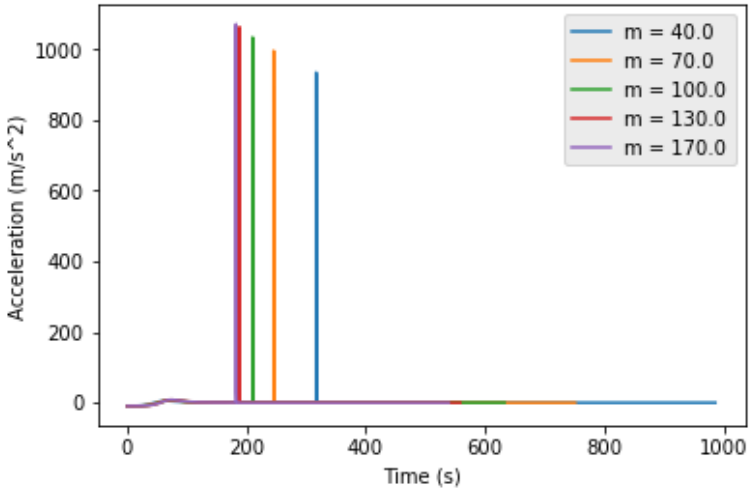

Figure: Acceleration vs time

The masses used were 40, 70, 100, 130, and 170 kg. The resulting final velocities were -3.60, -4.76, -5.69, -6.49, and -6.74 m/s, respectively.

Sensitivity to Launch Height

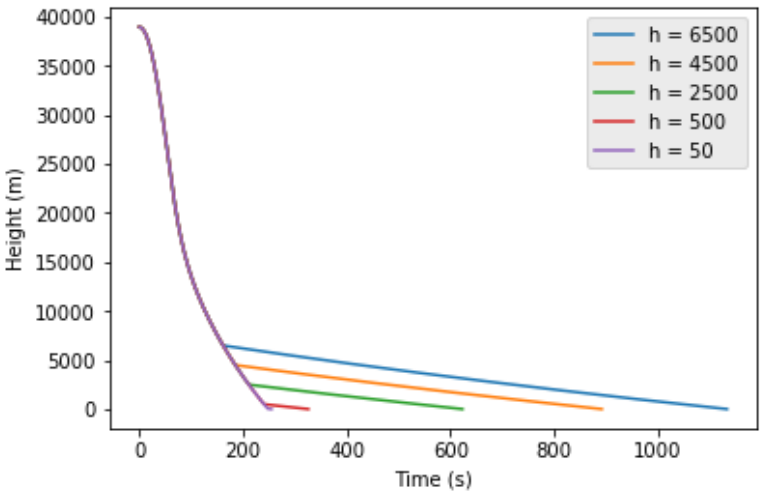

Figure: Position vs time

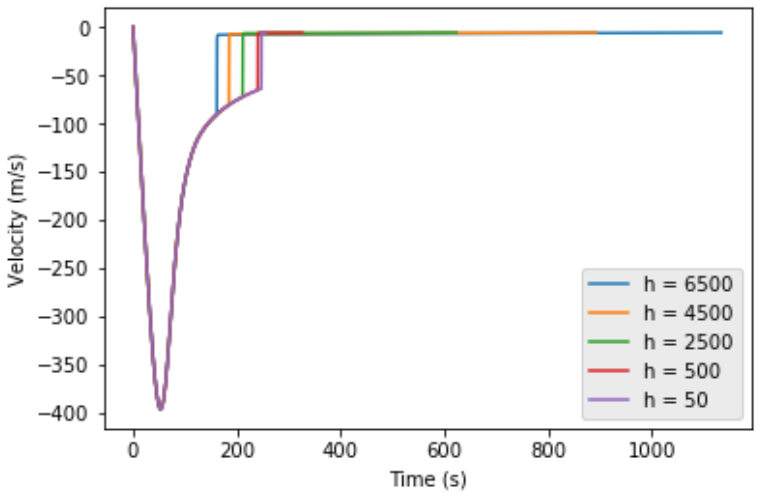

Figure: Velocity vs time

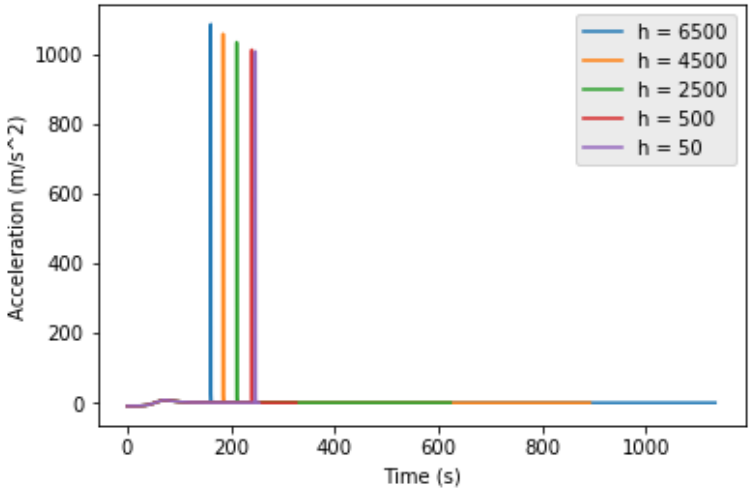

Figure: Acceleration vs time

The launch heights used were 6500, 4500, 2500, 500, and 50 m. The resulting final velocities were all -5.69 m/s.

Using RK4 to Model Felix Baumgartner's 38969 m Skydive

Team 3

Unit Testing

Felix Freefall

Felix With Parachute

RK4 Method

Sensitivity Tests

Summary

References

Summary

- RK4 model of ball in free fall
- Added drag
- Added gravity equation and barometric equation
- Added a parachute

References


Berberan-Santos, M. N., et al. Am. J. Phys. **65**, 5 (1997)


Shiflet, A. B. and Shiflet, G. W. *Science: Modeling and Simulation for the Sciences*. Princeton University Press (2014)